# Lifting under-achievement in Mathematics

Dan Murphy
Sabbatical Study 2015

## I met personally with:

| | |
|---|---|
| Charles Lovitt | Mathematics Consultant Melbourne |
| Prof. Doug Clarke | Australian Catholic Uni, Melbourne |
| Dr Barbara Clarke | Monash University, Melbourne |

## I read and studied the work of:

| | |
|---|---|
| Prof. Jo Boaler | Stanford Graduate School of Education |
| Dr Sue Gifford | Cambridge University |
| Stein & Engle et al | Pittsburg & Berkeley Universities USA |

Freudenthal Institute for Science and Mathematics Education (Netherlands)

# Part 1

## Analysing a closed task

- What does brain science tell us
- What does Mindset Psychology tell us
- What do International Assessments tell us
- What do international math experts say
- What does Charles Lovitt (my mentor) say

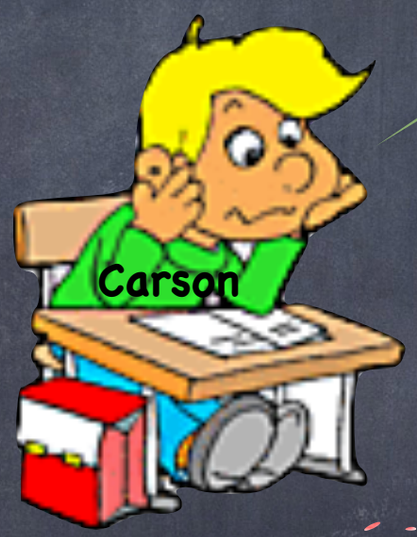
A math lesson which values memorisation and steps (procedural fluency).


- a closed task - you are either right or wrong
- commonly used in work sheets to teach to the middle of a year level

11

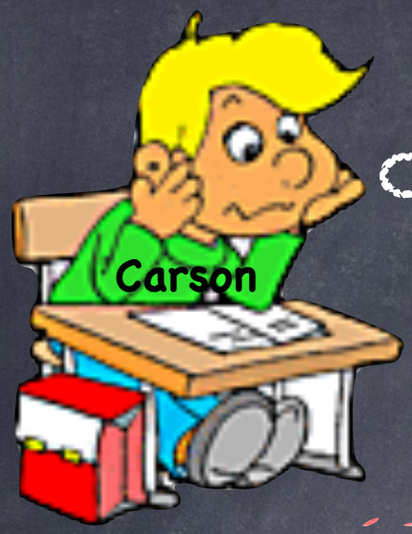
Good!
You got that right ...
as a reward you can do
50 more like that on
page 56.

13?



I didn't think he would know....

Wrong!
You can join the
Donkeys' Group and do
10 more like that!




I know I am no good at math .....

I am always in the Donkeys' Group

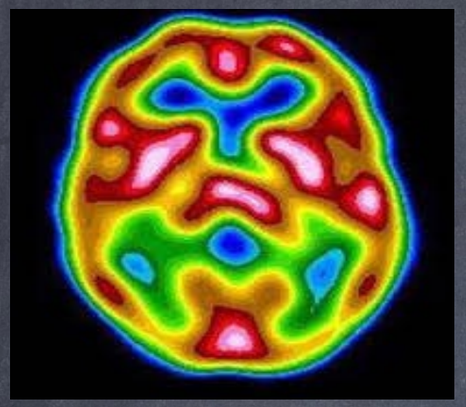
## Brain Science tells us:

- traditionally people thought intelligence was innate

- we now know that no one is born with a math brain

- with good experiences the brain has the capacity to change, rewire and grow

- students can grasp high level ideas but won't develop brain connections if they are given low level work and negative messages



- recent research shows that when students make a mistake in math, their brain grows, synapses fire, and connections are made

- when students do a page of exercises correctly - there is no brain growth - the brain barely lights up

- when children struggle with a problem - their brain lights up and grows connections

## Mindset Psychology says:

People with 'fixed mindsets' believe....

- you are either smart or you are not

- giving children lots of exercises so they get fast at recall and memorisation will make them good at mathematics

- children who struggle need lots of simple exercises and  shouldn't do higher level math

- when students with a fixed mindset make a mistake they believe they are not smart and give up -they avoid challenge and want easy work to do
- when children spend their time answering discrete questions with right or wrong answers it is very difficult to develop a growth mindset

## Math Experts say:

- part of the problem is the desperation of many parents to advance their children in math - pushing them to higher levels - faster and sooner

- mathematics is not a subject that requires fast thinking

- closed problems do not teach children to think, to use logic and reasoning, or to communicate

## International Assessments show:

- the lowest achieving of 13 million students on PISA tests were those who used memorisation - a set of steps to remember
- the highest achieving students were those who thought of math as a set of connected big ideas

Charles Lovitt says:

**Under-achievement is caused by …**

- the narrow closed-tasks teachers have favoured

- ability grouping with its negative connotations and the way it restricts coverage of the curriculum

- teachers' fear that problem solving ignores skills development and so they favour closed tasks

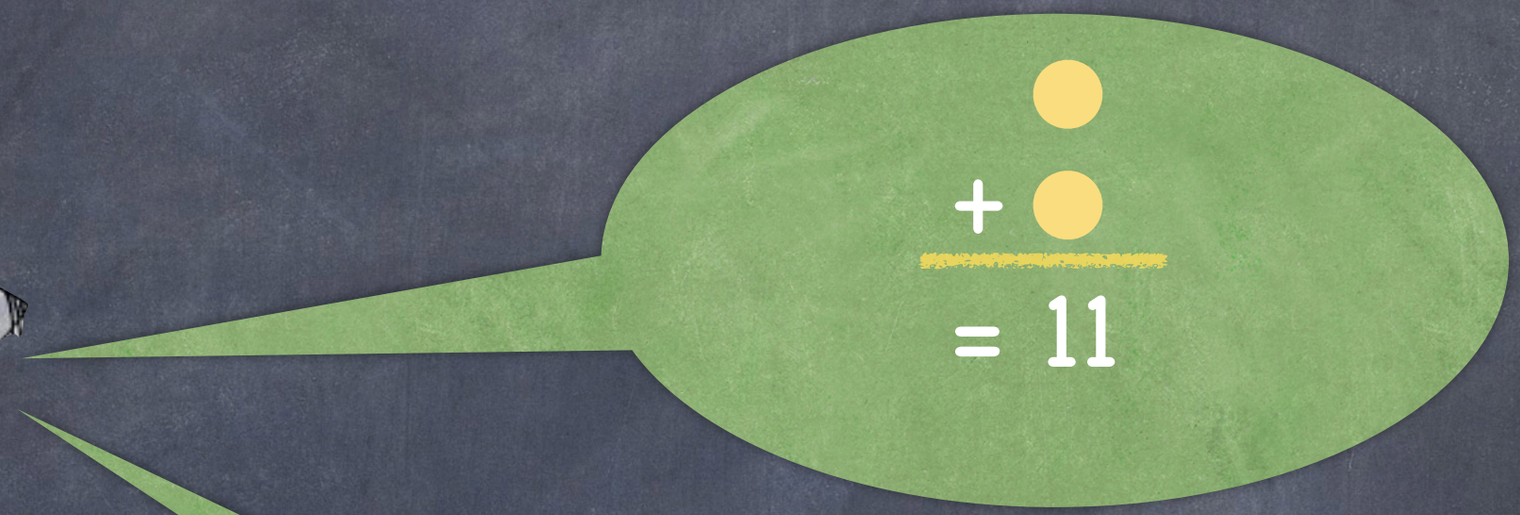
# Part 2

## Analysing a rich mathematical task

- What does brain science tell us
- What does Mindset Psychology tell us
- What do International Assessments tell us
- What do international math experts say
- What does Charles Lovitt (my mentor) say

# The common Japanese approach to a math lesson

Teachers collaborate to prepare tasks




Work
in your group to
find as many
solutions as you
can

In small groups try this task.

# Working like a mathematician.

How many solutions are there?
How do you know you have got them all?

A problem never ends... there is always more!

What if the answer was 12?

In small groups try this task.

What if the answer is …

13?

20?

43?

100?

n?

Can you develop a rule for this?

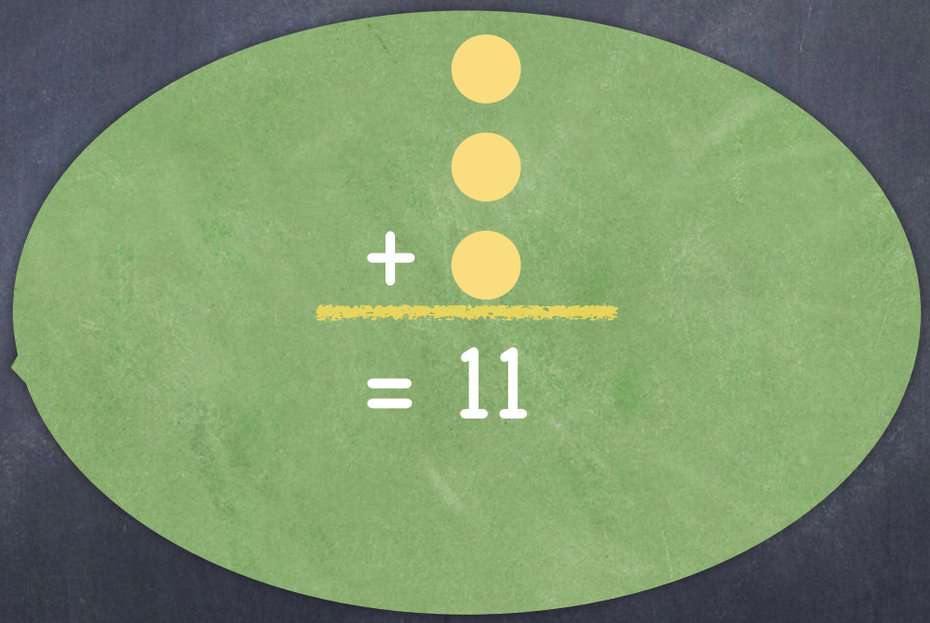
| | | | | | | |
|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 20 | 43 | 100 | n |
| 6 | 7 | 7 | 11 | 22 | 51 | |

Develop the algebra..... oops! - not so predictable

rule (if n is even) $n/2 + 1$

rule (if n is odd) $\frac{n + 1}{2}$

What if there are 3 numbers?



What if there are 4 numbers?

+

= 11

The (n,2) rule when:

(n is even) $n/2 + 1$

(n is odd) $\frac{n + 1}{2}$

What is the (n,3) rule?

What is the (n,4) rule?

What is the (n,$x$) rule?

This demonstrates - there is always more to a problem!

**The history:**

This problem was a most famous problem for 200 years.

It was finally solved by an Indian man.

## Brain Science tells us:

- with the plasticity of the brain - ability and intelligence grow with effort and practice

- children's brains light up when they are faced with a challenging problem

- a child's brain does not light up when doing meaningless memorisation tasks

- it is working on the complex that allows the brain to develop

## Mindset Psychology says:

New findings about brain development and mindset psychology are inextricably linked

- the plasticity of the brain is now well known yet ignored by the fixed mindset thinking of schools

- a child's mindset for learning and a teacher's mindset for teaching is critical and makes a difference

- the damaging effects of ability grouping on a child's mindset creates failure

- deliberate Mindset intervention resulted in a gain in achievement for African-Americans

- think what this approach could do for Maori/Pasifika in a NZ context?

- think how Winchester's ecology of learning and multiple intelligences theory... along with recent mindset and brain science could improve learning

## Ability Grouping

- reduces achievement overall

- particularly for high achieving girls who fear failure when in placed top group

- high level content is taught to only some children

- discourages children by communicating ... only some are high achievers and ability is fixed

**Interestingly…**

- average & below average children achieve higher levels in mixed ability classes ….

  and …..

- high achievers achieve the same in either setting!

## Math Experts say:


- tasks that are particularly valuable are those that have a low floor and a high ceiling

- anyone can access them, but they can be taken to very high levels

- math classes have valued one type of learner - children who can memorise well and calculate fast

## International Assessments show that:

- children who are taught memorisation techniques are the lowest performers in OECD's Programme for International Student Assessment (PISA) tests
- the highest performers are in the Pacific rim countries -and particularly Japan and China - where collaborative problem solving is encouraged and assessment de-emphasised

**Charles Lovitt says:**

Features of a good math task:

- multiple entry and multiple exit points
- the starting point is simple enough for all to enter
- has a story shell or history - a context
- no task is ever finished
- caters for the 7-year gap in classes
- it develops skills, thinking & reasoning, and communication

Another example to consider.

# A typical work sheet closed task

Find the average of these 3 numbers?
11,12,13

11
12
13
= 36 / 3

= 12

# Open Ended approach

In small groups try this task.

?

Find 3 numbers that make an average of 12?

- How many ways are there?
- How do you know you have got them all?

Answer:

There are 127 ways to make an average of 12

The task is....

- open ended
- all children can enter
- 'what if' can be applied
- plenty of skill practice provided

An interesting dilemma

Many schools introducing MLE's are developing the concept of '**student agency**'.

- teachers make their learning intentions (specific objectives) visible on the wall
- children rank themselves against each objective on the wall in a public display
- teachers plan workshops to meet indicated needs
- children opt in to workshops (children driving their learning = student agency)

- this model suits the 'numeracy type' programme where learning is broken into small measurable segments

- but... numeracy advisers are now promoting the problem solving approach ...

- ...and schools who want 'student agency' are finding it doesn't fit.....

- ..... because problem solving is more holistic, it can't be broken into bits when **children are working like mathematicians** (see next slide for details)

- learning includes ....skills, logic & reasoning , and communication - not just procedural fluency

# Working Mathematically

## Learning to Work like a Mathematician

*First give me an interesting problem.*

### When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

### Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if …?
- How many solutions are there?
- How will I know when I have found them all?

### When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

### A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Break the problem into smaller parts
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- ...

*If one way doesn't work I just start again another way.*

## Snippets from the experts:

- the on-going influence of the Cockcroft Report
- what Trends in International Mathematics and Science Study (TIMSS) tells us
- Japan once again points the way
- some advice to teachers

# Cockcroft Report UK

- this famous report was written by Professor Cockcroft following an investigation into the failure of mathematics teaching in the UK

- it continues to be highly regarded by academics

In the now-famous Paragraph 243 = ($3^5$) of the Report Cockcroft recommended that mathematics teaching at all levels should include opportunities for:

- exposition by the teacher
- discussion between teacher and students, and between students themselves
- appropriate practical work

- consolidation and practice of fundamental skills and routines

- problem solving, including the application of mathematics to everyday situations

- investigational work

Cockcroft reported that in any given class level there is a 7-year gap that teachers have to cater for.

5.5 yrs ..... **9 year old class** ..... 12.5 yrs

Typically work sheets teach to the middle

- this bores bright children
- leaves slow children behind

so ... to achieve this we need problem solving and open-ended investigations

Cockcroft identified the mathematical needs of adults as being:

- ability to read numbers
- to count
- to tell the time
- to pay for purchases and give change
- to understand timetables, simple graphs and charts
- perform sensible estimations and approximations

A study of countries success in Trends in International Mathematics and Science Study (TIMSS) - especially Japan & China found...

- a lesson focuses around a single problem
- teacher is a knowledgeable guide rather than dispenser of information
- children encouraged to explain and evaluate solutions
- lessons focus on problem solving rather than mastery of facts and procedures

**Japan has a strong belief that…**

…children should not be subjected to the measuring of their capabilities or aptitudes….

…nor the subsequent remediation or acceleration during their 9 years of compulsory education…

# Teachers need to….


- stop frequent timed testing
- make children feel good about mistakes & comfortable with struggle
- replace grades with constant feedback
- de-emphasise speed so students can think slowly and deeply
- remove groupings that transmit 'fixed mindset' messages

https://www.youtube.com/watch?v=4Xi6pLo58g4

Task:

In small groups summarise on a chart the key messages of this presentation and beside each message list possible actions we can take to succeed at it!